

# Baryon octupole moments\*

A. J. Buchmann<sup>1†</sup> and E. M. Henley<sup>2‡</sup>

<sup>1</sup>*Institut für Theoretische Physik, Universität Tübingen  
Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

<sup>2</sup>*Department of Physics and Institute for Nuclear Theory, Box 351560,  
University of Washington, Seattle, WA 98195, U.S.A.*

We report on a calculation of higher electromagnetic multipole moments of baryons in a non-covariant quark model approach. The employed method is based on the underlying spin-flavor symmetry of the strong interaction and its breaking. We present results on magnetic octupole moments of decuplet baryons and discuss their implications.

PACS numbers: 13.40.Em, 14.20.-c

## I. INTRODUCTION

Electromagnetic multipole moments of baryons are interesting observables. They are directly connected with the spatial charge and current distributions in baryons, and thus contain fundamental information about their internal structure and geometric shape.

For example, recent electron-proton and photon-proton scattering experiments, exciting the lowest lying nucleon resonance  $\Delta^+(1232)$  have provided evidence for a nonzero quadrupole moment  $Q_{p \rightarrow \Delta^+} \approx -0.08 \text{ fm}^2$  [1, 2, 3] and hence for a nonspherical charge distribution in baryons. The measured sign and magnitude are in agreement with the quark model prediction  $Q_{p \rightarrow \Delta^+} = r_n^2/\sqrt{2}$ , where  $r_n^2$  is the neutron charge radius [4]. It has been suggested that a transition quadrupole moment of this sign arises because the proton has a prolate and the  $\Delta^+$  an oblate charge distribution [5]. For a recent review see Ref. [6].

While there is a large body of literature on baryon magnetic dipole moments, relatively little is known about the next higher multipole moments, that is the magnetic octupole moments  $\Omega$  of decuplet baryons [7]. Presently, we neither know the sign nor the size of these moments. This information is needed to reveal further details of the current distribution in baryons beyond those available from the magnetic dipole moment [8].

The purpose of this paper is to calculate the magnetic octupole moments of baryons and to draw some conclusions concerning the shape of their spatial current distributions.

## II. METHOD

We use a general parametrization (GP) method developed by Morpurgo [9], which incorporates SU(6) symmetry and its breaking similar to the  $1/N_c$  expansion.

The basic idea is to *formally* define, for the observable at hand, a QCD operator  $\Omega$  and QCD eigenstates  $|B\rangle$  expressed explicitly in terms of quarks and gluons. The corresponding matrix elements can, with the help of the unitary operator  $V$ , be reduced to an evaluation in a basis of pure three-quark states  $|\Phi_B\rangle$  with orbital angular momentum  $L = 0$

$$\langle B|\Omega|B\rangle = \langle \Phi_B|V^\dagger \Omega V|\Phi_B\rangle = \langle W_B|\tilde{\Omega}|W_B\rangle. \quad (1)$$

The spin-flavor wave functions contained in  $|\Phi_B\rangle$  are denoted by  $|W_B\rangle$ . The operator  $V$  dresses the pure three-quark states with  $q\bar{q}$  components and gluons and thereby generates the exact QCD eigenstates  $|B\rangle$ .

One then writes the most general expression for the operator  $\tilde{\Omega}$  that is compatible with the space-time and inner QCD symmetries. Generally, this is a sum of one-, two-, and three-quark operators in spin-flavor space multiplied by *a priori* unknown constants which parametrize the orbital and color space matrix elements. Empirically, a hierarchy in the importance of one-, two-, and three-quark operators is found. This fact can be understood in the  $1/N_c$  expansion where two- and three-quark operators describing second and third order SU(6) symmetry breaking are usually suppressed by powers of  $1/N_c$  and  $1/N_c^2$  respectively compared to one-quark operators associated with first order symmetry breaking [10]. The GP method has recently been used to calculate charge radii and quadrupole moments of baryons [11, 12].

## III. CALCULATION

The magnetic octupole moment operator  $\Omega$  usually given in units  $[\text{fm}^2 \mu_N]$  and normalized as in Ref. [13] can be written as

$$\Omega = \frac{3}{8} \int d\mathbf{r}^3 (3z^2 - r^2) (\mathbf{r} \times \mathbf{J}(\mathbf{r}))_z, \quad (2)$$

where  $\mathbf{J}(\mathbf{r})$  is the spatial current density and  $\mu_N$  the nuclear magneton. This definition is analogous to the one for the charge quadrupole moment [5] if the magnetic moment density  $(\mathbf{r} \times \mathbf{J}(\mathbf{r}))_z$  is replaced by the charge density  $\rho(\mathbf{r})$ . Thus, the magnetic octupole moment measures the

\*published in: Eur. Phys. J. A **35**, 267 (2008).

<sup>†</sup>Electronic address: alfons.buchmann@uni-tuebingen.de

<sup>‡</sup>Electronic address: henley@u.washington.edu

deviation of the spatial magnetic moment distribution from spherical symmetry. More specifically, for a prolate (cigar-shaped) magnetic moment distribution  $\Omega > 0$ , while for an oblate (pancake-shaped) magnetic moment distribution  $\Omega < 0$ . We also see from Eq.(2) that the typical size of a magnetic octupole moment is

$$\Omega \simeq r^2 \mu \quad (3)$$

where  $\mu$  is the magnetic moment and  $r^2$  a size parameter related to the quadrupole moment of the system.

To construct an octupole moment operator  $\tilde{\Omega}$  in spin-flavor space along the lines outlined in sect. 2 we need a tensor of rank 3 in spin space. Clearly, this operator must involve the Pauli spin matrices of three different quarks [14] and can be built in two different ways. First, we can construct it from a three-body quadrupole moment operator multiplied by the spin of the third quark,

$$\tilde{\Omega}_{[3]} = C \sum_{i \neq j \neq k}^3 e_k (3\sigma_{iz}\sigma_{jz} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \boldsymbol{\sigma}_k, \quad (4)$$

where  $C$  is a constant and  $e_k = (1 + 3\tau_{kz})/6$  is the charge of the  $k$ -th quark. The  $z$ -component of the Pauli spin (isospin) matrix  $\boldsymbol{\sigma}_i$  ( $\boldsymbol{\tau}_i$ ) is denoted by  $\sigma_{iz}$  ( $\tau_{iz}$ ). Second, we can build it from a two-body quadrupole moment operator by replacing  $e_k$  by  $e_i$  in Eq.(4). Thus, it appears that there are two different operator structures and corresponding GP constants to be determined from experiment. However, from the point of view of broken SU(6) spin-flavor symmetry [15], there is a unique three-body operator containing a rank 3 spin tensor [16].

The magnetic octupole moments  $\Omega_{B^*}$  are then obtained by sandwiching the operator in Eq.(4) between the three-quark spin-flavor wave functions  $|W_{B^*}\rangle$ . For example, for  $\Delta(1232)$  baryons one obtains

$$\Omega_{\Delta} = \langle W_{\Delta} | \tilde{\Omega}_{[3]} | W_{\Delta} \rangle = 4C q_{\Delta}, \quad (5)$$

where  $q_{\Delta}$  is the  $\Delta$  charge. Similarly, the magnetic octupole moments for the other decuplet baryons are calculated. In this way Morpurgo's method yields an efficient parameterization of baryon octupole moments in terms of just one unknown parameter  $C$ .

#### IV. RESULTS

In the second column of Table I we show our results for the decuplet octupole moments expressed in terms of the GP constant  $C$  assuming that SU(3)-flavor symmetry is only broken by the electric charge operator as in Eq.(4). We observe that in this limit the magnetic octupole moments are proportional to the baryon charge. In order to estimate the degree of SU(3) flavor symmetry breaking beyond first order, we replace the spin-spin terms in Eq.(4) by expressions with a ‘‘cubic’’ quark mass dependence

$$\sigma_i \sigma_j \rightarrow \sigma_i \sigma_j m_u^3 / (m_i^2 m_j).$$

TABLE I: Magnetic octupole moments of decuplet baryons. Second column: SU(3) flavor symmetry limit ( $r = 1$ ). Third column: with flavor symmetry breaking ( $r \neq 1$ ).

|               | $\Omega_{B^*}(r = 1)$ | $\Omega_{B^*}(r \neq 1)$ |
|---------------|-----------------------|--------------------------|
| $\Delta^-$    | $-4C$                 | $-4C$                    |
| $\Delta^0$    | $0$                   | $0$                      |
| $\Delta^+$    | $4C$                  | $4C$                     |
| $\Delta^{++}$ | $8C$                  | $8C$                     |
| $\Sigma^{*-}$ | $-4C$                 | $-4C(1 + r + r^2)/3$     |
| $\Sigma^{*0}$ | $0$                   | $-2C(2 - r - r^2)/3$     |
| $\Sigma^{*+}$ | $4C$                  | $-4C(1 - 2r - 2r^2)/3$   |
| $\Xi^{*-}$    | $-4C$                 | $-4C(r + r^2 + r^3)/3$   |
| $\Xi^{*0}$    | $0$                   | $-4C(r + r^2 - 2r^3)/3$  |
| $\Omega^-$    | $-4C$                 | $-4C r^3$                |

This replacement is motivated by the flavor dependence of the gluon exchange current diagram [12]. Flavor symmetry breaking is then characterized by the ratio  $r = m_u/m_s$  where  $m_u$  and  $m_s$  denote the up and strange quark masses. This leads to analytic expressions for the magnetic octupole moments  $\Omega_{B^*}$  containing terms up to third order in  $r$  as shown in the third column of Table I.

Because the 10 diagonal octupole moments can be expressed in terms of only one constant  $C$ , there must be 9 relations between them. Given the analytical expressions in Table I it is straightforward to verify that the following relations hold

$$0 = \Omega_{\Delta^-} + \Omega_{\Delta^+}, \quad (6a)$$

$$0 = \Omega_{\Delta^0}, \quad (6b)$$

$$0 = 2\Omega_{\Delta^-} + \Omega_{\Delta^{++}}, \quad (6c)$$

$$0 = \Omega_{\Sigma^{*-}} - 2\Omega_{\Sigma^{*0}} + \Omega_{\Sigma^{*+}}, \quad (6d)$$

$$0 = 3(\Omega_{\Xi^{*-}} - \Omega_{\Sigma^{*-}}) - (\Omega_{\Omega^-} - \Omega_{\Delta^-}) \quad (6e)$$

$$0 = (\Omega_{\Xi^{*0}} + 2\Omega_{\Xi^{*-}}) + (\Omega_{\Sigma^{*+}} - \Omega_{\Sigma^{*-}}) \quad (6f)$$

$$0 = \frac{1}{3}(1 + r + r^2)\Omega_{\Delta^+} + \Omega_{\Sigma^{*-}}, \quad (6g)$$

$$0 = r\Omega_{\Sigma^{*-}} - \Omega_{\Xi^{*-}}, \quad (6h)$$

$$0 = r^3\Omega_{\Delta^-} - \Omega_{\Omega^-}. \quad (6i)$$

The first six relations do not depend on the flavor symmetry breaking parameter  $r$  and hold irrespective of the order of SU(3) symmetry breaking. In fact, Eqs.(6a-6d) are already a consequence of the assumed SU(2) isospin symmetry of strong interactions. Eq.(6e) is the octupole moment counterpart of the ‘‘equal spacing rule’’ for decuplet masses. Other combinations of the expressions in Table I can be written down if desirable.

To obtain an estimate for  $\Omega_{\Delta^+}$  we use the pion cloud model [5] where the  $\Delta^+$  wave function without bare  $\Delta$  and for maximal spin projection is written as

$$|\Delta^+ J_z = \frac{3}{2}\rangle = \beta' \left( \sqrt{\frac{1}{3}} |n' \pi^+\rangle + \sqrt{\frac{2}{3}} |p' \pi^0\rangle \right) | \uparrow Y_1^1 \rangle. \quad (7)$$

TABLE II: Numerical values for magnetic octupole moments of decuplet baryons in  $[\text{fm}^3]$  using Table I with  $C = -0.003$ . Second column: SU(3) flavor symmetry limit ( $r = 1$ ). Third column: with SU(3) flavor symmetry breaking ( $r = 0.6$ ).

|               | $\Omega_{B^*}(r = 1)$ | $\Omega_{B^*}(r = 0.6)$ |
|---------------|-----------------------|-------------------------|
| $\Delta^-$    | 0.012                 | 0.012                   |
| $\Delta^0$    | 0                     | 0                       |
| $\Delta^+$    | -0.012                | -0.012                  |
| $\Delta^{++}$ | -0.024                | -0.024                  |
| $\Sigma^{*-}$ | 0.012                 | 0.008                   |
| $\Sigma^{*0}$ | 0                     | 0.002                   |
| $\Sigma^{*+}$ | -0.012                | -0.004                  |
| $\Xi^{*-}$    | 0.012                 | 0.005                   |
| $\Xi^{*0}$    | 0                     | 0.002                   |
| $\Omega^-$    | 0.012                 | 0.003                   |

In this model the magnetic octupole moment operator is a product of a quadrupole operator in pion variables and a magnetic moment operator in nucleon variables

$$\Omega_{\pi N} = \sqrt{\frac{16\pi}{5}} r_\pi^2 Y_0^2(\mathbf{r}_\pi) \mu_N \tau_z^N \sigma_z^N. \quad (8)$$

Here, the spin-isospin structure of  $\Omega_{\pi N}$  is inferred from the  $\gamma\pi N$  and  $\gamma\pi$  currents of the static pion-nucleon model [17].

With these expressions the  $\Delta^+$  magnetic octupole moment is readily calculated [5]

$$\Omega_{\Delta^+} = -\frac{2}{15} \beta^2 r_\pi^2 \mu_N = Q_{\Delta^+} \mu_N = r_n^2 \mu_N, \quad (9)$$

where  $Q_{\Delta^+}$  is the  $\Delta^+$  quadrupole moment and  $r_n^2$  the

neutron charge radius. With the experimental value of the latter and  $\mu_N$  expressed in  $[\text{fm}]$  one gets  $\Omega_{\Delta^+} = -0.012 \text{ fm}^3$ . The negative value of  $\Omega$  implies that the magnetic moment distribution in the  $\Delta^+$  is oblate and hence has the same geometric shape as the charge distribution. Numerical values for other baryon octupole moments can now be obtained using Eq.(5) and the expressions in Table I. These are listed in Table II.

## V. SUMMARY

A general parameterization method based on SU(6) spin-flavor symmetry and its breaking has been used to calculate baryon octupole moments in a non-covariant quark model approach. We have constructed the magnetic octupole moment operator in spin-flavor space and have shown that it contains only third order symmetry breaking three-quark currents. We have then calculated analytical expressions for the octupole moments of decuplet baryons and derived certain relations between them.

To draw a first conclusion concerning the spatial shape of the magnetic moment distribution in baryons we have estimated the magnetic octupole moment of the  $\Delta^+$  in the pion cloud model. Our result can be expressed as the product of the  $\Delta^+$  quadrupole moment and the nuclear magneton. This means that the magnetic moment distribution in the  $\Delta^+$  is oblate and hence has the same geometric shape as the charge distribution. Numerical results have also been derived for other decuplet baryons.

It would be interesting to calculate the magnetic octupole moments in other models in order to check whether our finding of an oblate magnetic moment distribution in decuplet baryons can be confirmed.

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